Random discontinuous perturbations of NLS

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Abstract:

We consider the nonlinear Schrodinger equation [1] driven by a certain generalized "Dirac random comb" of the form

$$iu_t + u_{xx} + 2|u|^2 u = i\Big(-\Gamma u + \sum_n \big(e^{-\gamma_n} - 1\big)\delta(t - t_n)u(t_n^-, x)\Big) \tag{1}$$

where $\Gamma \in R^+$ is the normalized damping coefficient and $t_n > t_{n-1}$ and $\gamma_n > 0$ (jump positions and magnitudes) are certain sequences of random numbers. Such a perturbation incorporates the possibility of sudden changes in the field stemming from inhomogeneities in the media as might occur for an optical pulse propagating in a nonlinear optical fiber [2,3] which has impurities at certain random locations t_n at which the amplitude decreases from the "input" value $u(t_n^-, x)$ to an "output" value $e^{-\gamma_n}u(t_n^-, x)$.

If $\Gamma = 0$ we show that the resulting equation can be piecewise related to the unperturbed NLS equation and show how to solve the initial value problem in a global way. The construction involves the classical Poisson jump-process [4]. Even though the solution itself has jump discontinuities in time we show that a complete set of global conserved quantities exists for all time.

In the general case $\Gamma \neq 0$ the resulting equation is no longer integrable by IST ([5,1]). Nevertheless we determine the random evolution of several physically relevant quantities under the natural assumptions (i) $\delta_n \equiv t_n - t_{n-1} > 0$ is a sequence of independent, identically distributed random variables (iidrv), (ii) γ_n is a sequence of iidrv, (iii) δ_n is independent of γ_m and (iv) given that there are a fixed number n of jumps $t_1 < \dots t_n$ on $[0, t]$ then they are uniformly distributed on the interval. We show that their average value decreases exponentially in time due to the "impurities". We next formulate a linear integral equation that the mean half-life of the field Energy satisfies and, by means of a Laplace transform, find the solution.

References:

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